

MATH 441: Mock in-class Quiz

Duration: 50 minutes

Please note:

- The mock in-class quiz contains four problems, for a total of 50 points.
- The material covered is Chapter 2 of the notes (first-order ODEs).
- Write your solutions on the blank pages provided with the in-class quiz.
- The mock in-class quiz is optional. It will earn you 1% bonus point on your final grade if you attempt the in-class quiz.
- I suggest you spend about one minute per point.

Problem 1 (12 points)

Consider the initial value problem

$$y'(t) = -2te^{-y(t)}, \quad y(0) = 0.$$

- Solve the problem explicitly for $y(t)$ using separation of variables.
- Give the largest open interval containing $t = 0$ on which your solution is defined, and compute the limit of $y(t)$ at each endpoint of that interval.

Problem 2 (15 points)

Consider the initial value problem

$$y'(t) + ty(t) = t, \quad y(0) = 2.$$

- Solve the associated homogeneous ODE $y'(t) + ty(t) = 0$.
- Solve the initial value problem by variation of constants.
- Verify that the constant function $y_p \equiv 1$ solves the ODE. Use this observation together with your answer to a) to recover the answer to b) without computing an integral.

Problem 3 (13 points)

Consider the autonomous ODE

$$\dot{x} = x(1 - x)(x + 2).$$

- a) Find all fixed points and sketch the graph of $\dot{x}$ against x , marking the direction of the flow on the x -axis.
- b) Classify each fixed point as stable or unstable.
- c) Determine $\lim_{t \rightarrow \infty} x(t)$ for each of the initial values $x(0) = -1$, $x(0) = 1/2$ and $x(0) = 3$, without solving the ODE.

Problem 4 (10 points)

Consider the initial value problem

$$y'(t) = t + \sin(y(t)), \quad y(0) = 0.$$

Fix $T > 0$ and let $D_T = [0, T] \times [-C, C]$, where $C > 0$ is to be chosen.

- a) Verify assumptions (A1), (A2) and (A3) of the Picard–Lindelöf theorem on D_T , and give explicit values of the constants K and L .
- b) Choose C so that the theorem applies on $[0, T]$. Conclude that the initial value problem has exactly one continuously differentiable solution on $[0, \infty)$.

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