MATH 441: Assignment 1

Instructions: Submit your *hand-written* solutions in class by *Wednesday, September 9th, 11 AM*. I will *not* accept typed solutions unless you have an accommodation.

This assignment is meant to give you an idea of the material from differential and integral calculus you should be comfortable with in order to do well in MATH 441.

Problem 1

For (i)-(iii), compute the first and second derivatives dy/dx and d^2y/dx^2. For (iv)-(vi), use implicit differentiation to compute dy/dx.

(i) $y = -2x^3 + 8x - 1$.

(ii) $y = -3x^2$.

(iii) $y = -3e^{x^2}$.

(iv) $x^2 + y^2 = 2x$.

(v) $xy^2 = 3$.

(vi) $3xy + x^2y^2 = 10$.

Problem 2

(i) Solve for x: $1 = 10 - 10e^{-x}$.

(ii) Solve for x: $2 = 20 - 20\ln(x)$.

(iii) Show the equation $t = -\ln|(\csc(\pi/4) + \cot(\pi/4))/(\csc x + \cot x)|$ can be inverted in terms of x to get

$$x(t) = 2\arctan\left(\frac{e^t}{1+\sqrt{2}}\right).$$

Hint: You will need to use trigonometric identities to prove this.

Problem 3

Solve the following problems involving series. You may find it helpful to review the following concepts: the direct comparison Test, quotient test, Cauchy test, alternating series, and power series.

(i) Show that $\sum_{j=1}^{n} \left(\frac{1}{2}\right)^j = 1$.

(ii) Show that $\sum_{n=0}^{\infty} \frac{1}{n!} < \infty$.

(iii) Does the series $\sum_{n=0}^{\infty} \frac{n^3}{n!}$ converge or diverge? Explain your reasoning.

(iv) Does the series $\sum_{n=0}^{\infty} \frac{n^n}{n!}$ converge or diverge? Explain your reasoning.

(v) Does the series $\sum_{n=2}^{\infty} \frac{1}{(\ln n)^n}$ converge or diverge? Explain your reasoning.

(vi) Does the series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}$ converge or diverge? Explain your reasoning.

(vii) For what values of x does the series $\sum_{n=0}^{\infty} x^n$ converge, if any? Explain your reasoning.

Problem 4

Compute the following integrals:

(i) $\displaystyle\int \frac{-\sqrt{(a^2 - y^2)}}{y}\, dy.$

(ii) $\displaystyle\int x^{3/2}\, dx.$

(iii) $\displaystyle\int y e^{-y^2}\, dy.$

(iv) $\displaystyle\int_0^{\infty} e^{-st} \cos wt\, dt$, where $s,\, w \in \mathbb{R}$.

(v) $\displaystyle\int_{-t/2}^{t/2} \sqrt{\frac{t^2}{4} - x^2}\, dx,$ where $t > 0$. Hint: Use a hyperbolic trigonometric substitution to simplify the integral.

(iv) $\displaystyle\int \frac{a}{\sqrt{a^2 + s^2}}\, ds$, where $a > 0$.

(vii) $\displaystyle\int \frac{1}{y(K - y)}\, dy,$ where $K > 0$.

(viii) $\displaystyle\int \frac{1}{1 - y^2}\, dy.$