MATH 441: Homework 4

This homework is due on Wednesday April 1, in-class.

Readings

Boyce and diPrima: Sections 3.1-3.6, 6 and 7.

Problem 1

Compute the general solution to the following ODEs **using variation of constants:**

a) $ty''(t) - (t+1)y'(t) + y(t) = t^2$. **Hint:** The fundamental solutions are $1+t$ and e^t.

b) $y''(t) + 4y(t) = 2\tan(2t)$.

Problem 2

Compute the Laplace transform of the following functions:

$$\text{(i) } \sinh \omega t \quad \text{(ii) } \cosh \omega t \quad \text{(iii) } te^{ct} \quad \text{(iv) } t^2 e^{ct} \quad \text{(v) } \sin(\omega t).$$

Problem 3

Find the functions whose Laplace transforms are given by:

$$\text{(i) } \frac{s}{s^2+1} \quad \text{(ii) } \frac{1}{s^2+2s} \quad \text{(iii) } \frac{4s^3}{s^4-4} \quad \text{(iv) } \frac{1}{s(s^2+1)} \quad \text{(v) } \frac{s}{(s+1)^2}.$$

Problem 4

Solve the following initial value problems using the Laplace transform:

a) $3y''(t) - 3y'(t) + 4y(t) = e^{-t}$, where $y(0) = 1$, $y'(0) = -2$.

b) $y''(t) - 3y'(t) - 4y(t) = g(t)$, where

$$y(0) = y'(0) = 0 \text{ and } g(t) = \begin{cases} \cos(t) & 0 \leqslant t \leqslant \frac{\pi}{2}, \\ 0 & t \geqslant \frac{\pi}{2}. \end{cases}$$

c) $y''(t) + y(t) = g(t)$, where

$$y(0) = 1,\ y'(0) = 0 \text{ and } g(t) = \begin{cases} 0 & t < 3\pi, \\ 1 & t \geqslant 3\pi. \end{cases}$$

Problem 5

Let $\alpha > 0$. Consider the initial value problem

$$y''(t) - \alpha^2 y(t) = \sin(\alpha t), \text{ and } y(0) = y'(0) = 0.$$

a) Find the Green function for this problem.

b) Solve the problem using a). You do not have to evaluate the convolution integral.