MATH 441: Homework 3

This homework is due on Wednesday March 11, 5 PM.

Readings

Boyce and diPrima: Sections 3.1-3.6 and 6

Problem 1

Compute the general solution to each of the following ODEs:

a) $t^2y''(t) - ty'(t) + y(t) = 1$.

b) $t^2y''(t) - 3ty'(t) + 4y(t) = t$.

c) $t^2y''(t) - 2ty'(t) + 2y(t) = 3t^2 + \ln t$.

Problem 2

Compute the general solution to each of the following homogeneous ODEs:

a) $y''(t) + cy'(t) + \omega^2 y(t) = 0$, where $c > 0$ and $c^2 - 4\omega^2 < 0$.

b) $y^{(5)}(t) + 3y^{(4)}(t) - y^{(3)}(t) - 7y''(t) + 4y(t) = 0$. **Hint:** Verify that the characteristic polynomial of the ODE in part b) is $(\lambda-1)^2(\lambda+2)^2(\lambda+1)$ and then proceed as usual.

Problem 3

Solve the equation 'Septimi Gradus' due to Leonhard Euler (1743):

$$y^{(7)}(t) + y^{(5)}(t) + y^{(4)}(t) + y^{(3)}(t) + y^{(2)}(t) + y(t) = 0.$$

Hint: You may assume without proof that the characteristic polynomial is given by $(\lambda + 1)(\lambda^2 + \lambda + 1)(\lambda^2 - \lambda + 1)^2$.

Problem 4

Compute the general solutions to each of the following inhomogeneous ODEs:

a) $y''(t) - 4y(t) = te^{-t}$.

b) $y^{(3)}(t) + \omega^2 y'(t) = t^2(1 + \cos(\omega t))$.

Problem 5

Consider the following ODE:

$$y''(t) + \omega^2 y(t) = \sin(\omega_0 t),$$

where $\omega, \omega_0 \in \mathbb{R}$. Find the general solution to this ODE assuming:

a) $\omega \neq \omega_0$,

b) $\omega = \omega_0$.

You may use the results from Problem 2a. In addition, for each part, describe the behavior of $|y(t)|$ as $t \to +\infty$. What happens when $\omega = \omega_0$? The phenomenon you'll find is called **resonance**[1], and is practically very important!

[1] https://en.wikipedia.org/wiki/Resonance